PROPERTY RIGHTS AND GROWTH*

Ines Lindner

and

Holger Strulik

Department of Economics, University of Hamburg,

Von Melle Park 5, 20146 Hamburg

ilindner@hermes1.econ.uni-hamburg.de

strulik@hermes1.econ.uni-hamburg.de

We analyze the long-run equilibrium and adjustment dynamics in three popular models of economic growth when property rights are absent. The results are compared to the outcome in a corresponding economy with secure property rights. The main findings are that there exists a considerable gain in both, the level and the growth rate of consumption from establishing secure property rights, that economic performance without property rights worsens with increasing number of competing groups, and that the existence, or absence of property rights explains conditional convergence.

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1 Introduction

Standard growth literature assumes secure property rights. A large number of individuals supply capital and labor and in exchange receive factor income according to the marginal product of these factors. Secure factor income is guaranteed by secure property rights, a reasonable assumption for a fully developed economy situated in a democratic, constitutional state. A less developed economy which is characterized by a high degree of social conflict, a low degree of institutionalized or enforceable laws, or a high degree of political instability may be better approximated by the assumption that secure property rights are absent. In this paper we introduce missing property rights into three popular models of economic growth and compare the results with the corresponding results from standard models. The comparison provides an assessment of the importance of property rights for economic development and of the possible gain from establishing secure property rights.

Empirical studies which include a proxy for insecure property rights usually find it negatively correlated with economic growth, see e.g. Sala-i-Martin [16], Scully [17], Goldsmith [7] and Keefer and Knack [11]. In a recent survey Freeman and Lindauer [6] argue that missing property rights can be identified as the one major obstacle to growth for Sub Saharan Africa.

If the proxy for insecure property rights serves as an explanatory variable for economic growth then it serves also as an explanatory variable for conditional convergence. An economy with insecure property rights converges towards a different long-run state than an otherwise identical economy with secure property rights. In this paper we offer a theoretical explanation for conditional convergence. We show that individuals in an economy without property rights select a lower investment rate and – depending on the given production technology – approach either a lower level of consumption or a lower long-run growth rate than individuals in an economy with secure property rights. Growth is conditioned on investment but investment in turn is conditioned on the existence of property rights.

We model missing property rights by the assumption of a society of different groups in which all groups have the right to invest and the right to expropriate. The groups play a dynamic game of capital accumulation and expropriation. For a low number of groups we think of powerful political or ethnic groups and for a high number of groups we think of a society close to anarchy. In Section 2 we set up the framework with a general production function and calculate feedback Nash-strategies. In the remaining sections we introduce specific forms of the production function and discuss the results.

The Nash-equilibrium is not the only available solution. Benhabib and Rustichini [4] argue that social norms may develop which enable two players to reach a pareto-optimal equilibrium using trigger strategies. This way they calculate an upper bound for growth without property rights. Their approach can be understood as a complement to this paper.

In Section 3 we introduce missing property rights to the neoclassical growth model [5]. A special two-player case of this game has a long history in the economic literature. It is the game of capitalism as developed in [12] where one group has the right to invest and bears the risk to be (partly) expropriated by the second group. The second group in turn lacks the right to invest. Shimomura [18] shows how to solve the game in feedback strategies with nonlinear utility functions and convex technologies, which relates the game of capitalism close to the neoclassical growth model without property rights. Hence, Section 3 can also be understood as a generalization of Shimomura's game of capitalism. We show that the existence or absence of property rights explains conditional convergence in levels. Starting at the same initial state, individuals in an economy without property rights select a lower investment rate and converge towards a lower steady-state consumption level.

In the fourth section we assume linear strategies and introduce missing property rights to the linear growth model. This model has already been analyzed by Lane and Tornell [13] albeit with different conclusions. They focus on the so called voracity effect where a positive technology shock leads to lower growth. We show that an economy without property rights displays no voracity effect if an otherwise identical economy with secure property rights is capable of long-run growth. We then consider the effect of the number of competing groups on economic growth. An increase in the number of groups reduces the growth rate. If the number of groups becomes large an economy without property rights is not capable to grow with a positive rate.

Section 5 combines both models by the introduction of missing property rights in a convex model of growth [9]. We show that an economy without property rights adjusts towards a steady-state of lower growth than an otherwise identical economy with secure property rights. The result of conditional convergence arises because individuals in the economy without property rights select a lower investment ratio for any given capital productivity along the adjustment path.

The notation follows the presentation of the three models with secure property rights in [3]. In order to be brief, our analysis frequently refers to results displayed in this widespread textbook of growth theory.

2 The General Framework

The economy is populated by $n \geq 2$ homogenous groups. Each group $i = 1, 2, \ldots, n$ consists of a continuum $[0, 1]$ of agents with intertemporal utility of consumption, c_i, according to

$$\int_0^\infty \frac{c_i^{1-\theta} - 1}{1-\theta} \mathrm{e}^{-\rho t} \mathrm{d}t \ . \tag{1}$$

In (1) $\rho > 0$ denotes the time preference rate and $1/\theta > 0$ the intertemporal elasticity of substitution.

Using capital $k \geq 0$ a single output is produced via a production function f. There exist no property rights so that agents of each group are free to choose consumption and the evolution of k is given by

$$\dot{k} = f(k) - \delta k - \sum_{i=1}^{n} c_i \ , \tag{2}$$

where $\delta \geq 0$ denotes the rate of depreciation.

The production function f is twice continuously differentiable with $f(0) = 0$, $f' > 0$, $f'' \leq 0$.

Given $k(0) = k_0 \geq 0$ agents maximize (1) with respect to (2) using a feedback Nash-strategy, $c_i(k)$. Since all agents share symmetric utility functions and the same state equation, we confine the analysis to Nash-equilibria in symmetric strategies:

$$c_i = c \quad \text{for} \quad i = 1, \ldots, n \ . \tag{3}$$

We begin with constructing the Hamilton-Jacobi-Bellman equations for our differential game by applying Theorem 6.16 of Basar and Olsder.

THEOREM 2.1. *If a continuously differentiable function $V(k)$ can be found that satisfies*

$$\rho V(k) = (V'(k)^{(\theta-1)/\theta} - 1)/(1-\theta) + V'(k)[f(k) - \delta k - nV'(k)^{-1/\theta}] \ , \tag{4}$$

subject to the boundary condition

$$\lim_{t\to\infty} V(k(t)) \exp[-\rho t] = 0 \ , \tag{5}$$

where $k(t)$ is the nonnegative solution to $\dot{k} = f(k) - \delta k - nc(k)$, with $k(0) = k_0$ and

$$c(k) = V'(k)^{-1/\theta} \ , \tag{6}$$

then it generates a symmetric feedback Nash-equilibrium with the strategy of each player defined by (6).

Proof. Using the Hamiltonian functions defined by

$$H_i(k, c_1, ..., c_n, \lambda_i, t) := \frac{c_i^{1-\theta} - 1}{1-\theta} \exp[-\rho t] + \lambda_i[f(k) - \delta k - c_1 - ... - c_n] \tag{7}$$

the Hamilton-Jacobi-Bellman equations can be written as:

$$\begin{aligned} -\frac{\partial S_i(k,t)}{\partial t} &= \max_{c_i} H_i(k, c_1(k,t), ..., c_i, ..., c_n(k,t), \frac{\partial}{\partial k} S_i(k,t), t) \ , \\ c_j(k,t) &= \arg\max_{c_j} H_j(k, c_1(k,t), , , , , c_j, ..., c_n(k,t), \frac{\partial}{\partial k} S_j(k,t), t) \end{aligned} \tag{8}$$

for $i, j = 1, ..., n$ with boundary conditions

$$\lim_{t\to\infty} S_i(k(t), t) = 0, \qquad i = 1, ..., n \ , \tag{9}$$

and $k(t) \geq 0$ solves $\dot{k} = f(k) - \delta k - c_1(t) - ... - c_n(t)$ with $k(0) = k_0$.

If there are C^1-functions $S_1(k,t), ..., S_n(k,t)$ which satisfy (8) and (9), then they generate a feedback Nash-equilibrium by maximizing the Hamiltonians (8).

By setting $S(k,t) = V(k)\exp[-\rho t]$ solving eqs. (8) and (9) simplifies to solving the following system of ordinary differential equations:

$$\begin{aligned} \rho V_i(k) &= \max_{c_i} H_i(k, c_1(k), ..., c_i, ..., c_n(k), V_i'(k), 0), && (10) \\ c_j(k) &= \arg\max_{c_j} H_j(k, c_1(k), ..., c_j, ..., c_n(k), V_j'(k), 0) \ , && (11) \end{aligned}$$

with boundary conditions

$$\lim_{t\to\infty} V_i(k(t))\exp[-\rho t] = 0 \ . \tag{12}$$

Maximization of the Hamiltonians provides

$$c_i^{-\theta} = V_i'(k) \Leftrightarrow c_i = V_i'(k)^{-1/\theta} \ , \tag{13}$$

and (10) can be rewritten as

$$\rho V_i(k) = H_i(k, V_1'(k)^{-1/\theta}, ..., V_n'(k)^{-1/\theta}, V_i'(k), 0) \ . \tag{14}$$

For symmetric solutions, (12) and (14) simplify to

$$\rho V(k) = H(k, V'(k)^{-1/\theta}, ..., V'(k)^{-1/\theta}, V'(k), 0) \ , \tag{15}$$

$$\lim_{t\to\infty} V(k(t))\exp[-\rho t] = 0 \ , \tag{16}$$

which equals (4) and (5), and applying (13) provides (6). $\square$

Theorem 2.2. *Let c be a solution of*

$$c'(k) = \frac{[f'(k) - \delta - \rho]c(k)}{\theta[f(k) - \delta k - nc(k)] + (n-1)c(k)} \ , \tag{17}$$

with boundary condition

$$\lim_{t\to\infty} \int_{k0}^{k(t)} u'(c(y))\mathrm{d}y \exp[-\rho t] = 0 \ , \tag{18}$$

where $k(t)$ is the corresponding non-negative state-trajectory of

$$\dot{k} = f(k) - \delta k - nc(k), \qquad k(0) = k_0.$$

Then $(c_1, ..., c_n) = (c, ..., c)$ consitutes a symmetric feedback Nash-equilibrium.

Proof. Let $V(k)$ be defined as

$$V(k) := \int_{k0}^{k} u'(c(y))\mathrm{d}y + V(k_0) \ , \tag{19}$$

with $V(k_0)$ given by

$$V(k_0) = (1/\rho)\{u(c_0) + u'(c_0)[f(k_0) - \delta k_0 - nc_0]\} \ .$$

We verify that $V(k)$ is a solution to (4): Differentiating (4) with respect to k and substituting $u'(c) = V'(k)$ and $u''(c)c'(k) = V''(k)$ provides (17). Equation (19) is obtained by integrating (6). Insertion of $V(k)$ into (4) using $V'(k_0) = u'(c_0)$ yields the initial value $V(k_0)$ and (18) ensures that the transversality condition (5) holds. □

Since $c'(k) = \dot{c}/\dot{k}$, (17) can be decomposed into

$$\dot{c} = (f'(k) - \delta - \rho)c/\theta \ , \tag{20}$$

$$\dot{k} = f(k) - \delta k - nc + (n-1)c/\theta \ . \tag{21}$$

The ordinary differential equation system (20) and (21) bear a striking resemblance to the solution of the standard growth model. The decomposition provides two advantages which are exploited throughout the remainder of the paper: The problem can be solved for $c(k)$ with standard methods and it can be easily compared to the solution of the corresponding model with secure property rights. For these purposes, however, we have to specify the production function.

3 Property Rights and Growth: The Neoclassical Case

The neoclassical production function is assumed to be of the Cobb-Douglas type:

$$f(k) = Ak^{\alpha} \ , \ \ 0 < \alpha < 1 \ , \ \ A > 0 \ . \tag{22}$$

Insertion of (22) in (20) and (21) provides

$$\dot{c} = (\alpha A k^{\alpha-1} - \delta - \rho)c/\theta \ , \tag{23}$$

$$\dot{k} = Ak^{\alpha} - \delta k - nc + (n-1)c/\theta \ . \tag{24}$$

To assess the advantage of existing property rights we introduce an otherwise identical economy with secure property rights. This is an economy with a large number of firms operating on competitive markets and a continuum $[0, n]$ of price-taking consumers which follow the Ramsey rule (23). The aggregate budget constraint is obtained after inserting (22) into (2).

Theorem 3.1. *An economy with secure property rights converges towards the equilibrium*

$$k_p = k^{\star} = \left(\frac{\rho+\delta}{\alpha A}\right)^{1/(\alpha-1)} \ , \ \ c_p = \frac{Ak^{\star\alpha} - \delta k^{\star}}{n} \tag{25}$$

The proof is in [5] and in [3], Ch. 2.

We confine the analysis to the case where the economy without property rights is initially situated below the long-run equilibrium of an otherwise identical economy with secure property rights: $k(0) < k^{*}$.

Theorem 3.2. *If $\theta > (n-1)/n$, then an economy without property rights converges along a unique path towards an equilibrium level of consumption which falls short of the consumption level of an otherwise identical economy with secure property rights.*

Proof. Step 1: For $\theta > (n-1)/n$ system (23) and (24) has a unique positive equilibrium at

$$k^{\star} = \left(\frac{\rho+\delta}{\alpha A}\right)^{1/(\alpha-1)} , \ \ c^{\star} = \frac{Ak^{\star\alpha} - \delta k^{\star}}{n-(n-1)/\theta} \ . \tag{26}$$

The Jacobian determinant evaluated at $c^\star$, $k^\star$ is $\det J = (n-(n-1)/\theta)\alpha(\alpha-1)Ak^{\star\alpha-2}c^\star/\theta$ and negative for $\theta > (n-1)/n$. The equilibrium is a saddlepoint.

Step 2: Figure 1 displays the phase diagram, where the $\dot{k} = 0$ locus of (24) is given by $(Ak^\alpha - \delta k)/(n-(n-1)/\theta)$ and the $\dot{c} = 0$ locus is the vertical line at $k^\star$. All integral curves except the stable manifold can be excluded for violating the transversality condition.

Let k_1 denote the intersection of the stable manifold with the abscissa. Assume a capital stock $0 < k^c < k_1$ exists with $c(k^c) = 0$. Let the point in time when this happens be denoted by $\tau > 0$. Then from (13) it follows that $V_i'(k(\tau)) = \infty$ and $(\mathrm{d}/\mathrm{d}t)(V_i'(k)) = V_i''(k)\dot{k} < 0$ for $k < k^*$ so that $V_i'(k(t)) > V_i'(k(\tau))$ for all $t < \tau$ which is a contradiction to $V_i'(k(\tau))$ being infinite. Therefore, the stable manifold goes through the origin.

Step 3: The equilibrium of the economy is situated where the stable manifold intersects the *real* $\dot{k} = 0$ locus obtained from (2) with (22) as

$$\tilde{c}(k) = (Ak^\alpha - \delta k)/n \ , \tag{27}$$

with

$$c' = \frac{\alpha Ak^{\alpha-1} - \delta - \rho}{n-1} \ , \tag{28}$$

at the intersection. Since $c^\star > c_p$, consumption at an intersection $k^{\star\star}$ falls short of c_p, $c^{\star\star} < c_p$.

Suppose there are multiple intersections $k^\star > k_1 > k_2 > \ldots$. The stable manifold is located above the $\tilde{c}(k)$-curve at $k^\star$ so that at k_1:

$$c'(k_1) > \tilde{c}'(k_1) \ \Leftrightarrow\ \frac{\alpha Ak_1^{\alpha-1} - \delta - \rho}{n-1} > \frac{\alpha Ak_1^{\alpha-1} - \delta}{n} \ \Leftrightarrow\ \alpha Ak_1^{\alpha-1} > \delta + n\rho \ , \tag{29}$$

Hence, at k_2:

$$c'(k_2) < \tilde{c}'(k_2) \ \Leftrightarrow\ \frac{\alpha Ak_2^{\alpha-1} - \delta - \rho}{n-1} < \frac{\alpha Ak_2^{\alpha-1} - \delta}{n} \ \Leftrightarrow\ \alpha Ak_2^{\alpha-1} < \delta + n\rho \ , \tag{30}$$

and therefore $\alpha Ak_2^{\alpha-1} < \delta + n\rho < \alpha Ak_1^{\alpha-1}$, which contradicts the assumption $k_1 > k_2$. Hence, if an equilibrium exists, it is unique.

It remains to prove that a positive intersection exists. Since $c(k)$ is situated below the $\dot{k} = 0$ locus of (24), $(Ak^\alpha - \delta k - nc) + (n-1)c/\theta > 0$ for $k \in (0, k^\star)$ and hence

$$c' = \frac{(\alpha Ak^{\alpha-1} - \delta - \rho)c}{\theta(Ak^\alpha - \delta k - nc) + (n-1)c} < \frac{(\alpha Ak^{\alpha-1} - \delta)c}{\theta(Ak^\alpha - \delta k - nc) + (n-1)c} \ . \tag{31}$$

for $k \in (0, k^\star)$. Suppose $c(k) > \tilde{c}(k)$ for $k \in (0, k^\star)$. Then there exists an $\epsilon > 0$, $\epsilon < k^\star$ so that

$$c' = \frac{(\alpha A k^{\alpha-1} - \delta - \rho)c}{\theta(Ak^\alpha - \delta k - nc) + (n-1)c} > \tilde{c}' = \frac{\alpha A k^{\alpha-1} - \delta}{n} \tag{32}$$

for $k \in (0, \epsilon)$ and (31) implies

$$\frac{(\alpha A k^{\alpha-1} - \delta)c}{\theta(Ak^\alpha - \delta k - nc) + (n-1)c} > \frac{\alpha A k^{\alpha-1} - \delta}{n} \tag{33}$$

for $k \in (0, \epsilon)$. Since $\alpha A k^{\alpha-1} - \delta > 0$ for $k \in (0, \epsilon)$ it follows that

$$\frac{c}{\theta(Ak^\alpha - \delta k - nc) + (n-1)c} > \frac{1}{n} \tag{34}$$

for $k \in (0, \epsilon)$, and taking the limit

$$\lim_{k \to 0} \frac{c}{\theta(Ak^\alpha - \delta k - nc) + (n-1)c} = \frac{1}{n(1-\theta) - 1} \geq \frac{1}{n} \ , \tag{35}$$

which is a contradiction since the left hand side of the inequality condition is negative for $\theta > (n-1)/n$.

Hence, a unique positive equilibrium $k^{\star\star} < k^\star$ exists.

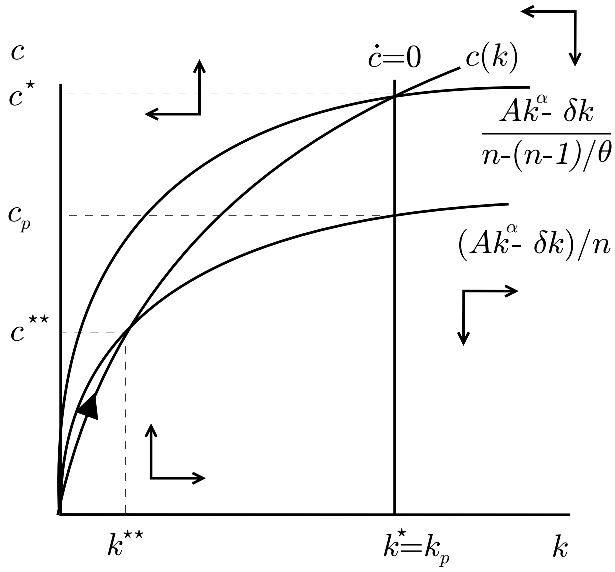
Step 4: Because $c^{\star\star}$ and $k^{\star\star}$ are positive and constant, the transversality condition (18) is fulfilled. □

The additional condition for existence of a feedback equilibrium requires that θ exceeds 0.5 for two competing groups and $\theta > 1$ for $n \to \infty$. Since estimation results as well as rules of thumb suggest that the intertemporal elasticity of substitution, $\sigma = 1/\theta$, is well below one, the additional condition is a mild one.[1]

The question remains whether the absence of property rights does *significantly* affect the performance of a developing economy. This question can only be answered numerically. Therefore, we parameterize the model and determine adjustment dynamics and steady-state consumption by means of backward integration.

[1] See [14], [8], see [15] for estimates of σ for developing countries. One could argue that interpreting powerful groups as rich oligarchs shifts the range of possible values upwards, since it has been shown that σ increases in wealth levels. Evidence for elasticities above one, however, is lacking. In a panel analysis Atkeson and Ogaki [1] estimate a σ around 0.8 for the richest Indian households.

Figure 1: Phase Diagram – Neoclassical Growth



In the first step we compute the stable manifold of (23) and (24) and obtain a numerical solution for the Nash-strategy $c(k)$. Let ϵ denote a small positive number close to the smallest computable number on the computer. Starting in $k^\star - \epsilon$, $c^\star - \epsilon$ we integrate the parameterized system (23) and (24) backwards in time using $k \approx \epsilon$ as termination criterion. Hence, we replace the inherently unstable boundary value problem by an inherently stable initial value problem which can be solved easily and accurately with standard methods[2]. With M denoting the number of executed integration steps the procedure provides a list of values for k and c and after reverting them we get the forward looking list of values $((k_j)_{j=1}^M, (c_j)_{j=1}^M)$ with $(k_M, c_M) \approx (k^\star, c^\star)$. From this list we use the first m elements, $((k_j)_{j=1}^m, (c_j)_{j=1}^m)$ with $f(k_m) - \delta k_m - nc_m \approx 0$ so that (k_m, c_m) is an approximation for the equilibrium of (2).

In the second step we use the *real* equation of motion obtained from (2) and (22) as $g(k) = Ak^\alpha - \delta k - nc$ and calculate the *real* time paths by setting $t = 0$ at k_0 and integrating

$$t_{j+1} - t_j = \int_{k(j)}^{k(j+1)} 1/g(k)\mathrm{d}k \tag{36}$$

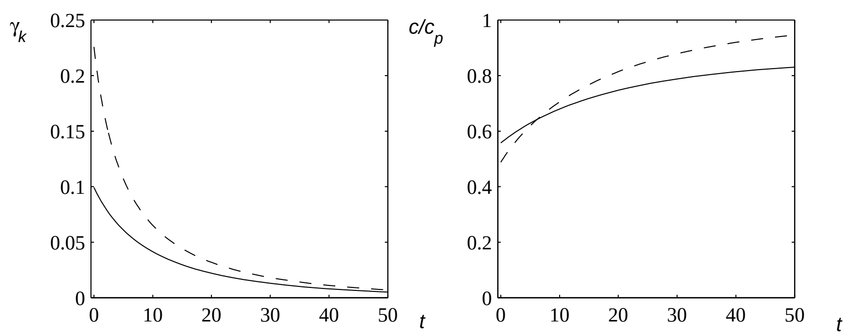
[2]We have used MATLAB's ODE45.

for the $i = 1, \ldots, m-1$. For that purpose we employ the trapezoidal rule.

We compare the result with the outcome in an otherwise identical economy with secure property rights, for which the time paths are also obtained by means of backward integration.

Figure 2 shows the result for a basic parameterization described below the Figure. The capital growth rate is denoted by γ_k. The consumption level is measured in terms of steady–state consumption of the economy with secure property rights. Solid lines represent the economy without property rights and dashed lines are the standard result for an economy with secure property rights. Without property rights people consume more in the first seven years as compared to people in an economy with secure property rights. The comparatively low investment rate results in convergence towards a long-run consumption level of 87 percent of an economy with secure property rights.

Figure 2: Adjustment Dynamics: Neoclassical Growth



Solid Lines: Without Property Rights, Dashed Lines: With Property Rights

$n = 2$, $\alpha = 0.3$, $\delta = 0.05$, $\rho = 0.02$, $\theta = 4$, $A = 1$

In Table 1 we compare steady-state consumption for a variety of parameterizations of the model. The table shows the consumption level per capita without property relative to consumption per capita in an otherwise identical economy with property rights. Relative consumption decreases sharply with increasing number of competing groups. If there are only two competing groups people may end up with 90 percent of consumption of an economy with secure property rights. The arrival of a third group, however, reduces this ratio to about 50 percent. If we define $n \to \infty$ as anarchy, the population converges to starvation if the society converges towards anarchy. This can also be seen from Figure 1 and (26) –(28). An increasing number of competing groups shifts $c^\star$ downwards and increases c' at the intersection point $k^{\star\star}$ thereby reducing $c^{\star\star}$.

TABLE 1

Steady-State Consumption Without Property Rights

Relative to Consumption With Property Rights

	$n = 2$	$n = 3$	$n = 5$	$n = 20$
Basic Scenario[a]	87	51	25	3.5
$\rho = 0.04$	82	47	22	3.0
$\alpha = 0.4$	83	47	22	2.6
$\delta = 0.1$	90	54	28	4.1

[a] $\alpha = 0.3$, $A = 1$, $\rho = 0.02$, $\theta = 4$, $\delta = 0.05$. Numbers in percent and rounded.

An increasing number of competing groups increases the possibility of being exploited and hence reduces the incentive to invest in the common stock. This can be seen by comparing capital productivities. For the basic parameterization the steady-state interest rate is 2 percent in an economy with secure property rights and equates the net marginal productivity of capital. In contrast, net marginal capital productivity is 7.9 percent in an economy without property rights and two competing groups. Generally, the result can be used as an explanation for the observance of low investment rates despite a small capital stock and high capital productivity in economies without secure property rights. Starting at the same initial state, individuals in an economy without property rights select a lower initial investment rate and converge towards a lower steady-state consumption level. Hence the model explains conditional convergence *in levels*, where the condition is the existence of secure property rights.

4 Linear Growth and the Voracity Effect

In this section we specify a linear technology

$$f(k) = Ak, \quad A > 0 \tag{37}$$

so that the economy has the potential for long-run growth. This model has already been analyzed in [13], albeit with different conclusions.

The corresponding economy with secure property rights is populated by a continuum $[0, n]$ of consumers acting according to the Ramsey-rule and the aggregate budget constraint obtained from insertion of (37) into (20), and (2), respectively.

THEOREM 4.1. *If*

$$\varphi > 0 \ , \tag{38}$$

$$A - \delta > \varphi \quad \Leftrightarrow A - \delta > \rho, \tag{39}$$

where φ is defined as

$$\varphi = \left[\frac{\theta - 1}{\theta}(A - \delta) + \frac{\rho}{\theta}\right] \ , \tag{40}$$

then an economy with secure property rights develops along a path of positive constant growth with

$$c = \frac{\varphi}{n} k \tag{41}$$

and

$$\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = (1/\theta)(A - \delta - \rho) \ . \tag{42}$$

The proof is in [3], Ch. 4.1.

If φ is too small, utility grows without bound and the solution for the utility maximization problem cannot be found. This is indicated by condition (38). If conditions for growth are too bad, a solution exists, but the implied growth rate is negative. This implication requires (39) to hold.

THEOREM 4.2. *If an economy with secure property rights has a positive growth path, and if*

$$\theta > \frac{n - 1}{n} \ , \tag{43}$$

then an otherwise identical economy without property rights has a Nash-equilibrium given by

$$c_i = c = \chi k, \quad \textit{where } \chi = \varphi \frac{\theta}{n\theta - (n-1)} \tag{44}$$

for $i = 1, \ldots, n$.

Furthermore, if

$$A - \delta > \varphi \frac{\theta n}{n\theta - (n-1)} \Leftrightarrow \frac{A-\delta}{n} > \rho \tag{45}$$

holds, then the economy develops along a path of positive constant growth with

$$\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{A - \delta - \rho n}{n\theta - (n-1)} \quad . \tag{46}$$

Proof. Insertion of (37) and $c(k) = \chi k$ into (17) provides χ in (44).

The transversality condition (18) is

$$\begin{aligned} 0 &= \lim_{t\to\infty} \int_{k0}^{k(t)} u'(c(y))\mathrm{d}y \exp[-\rho t] = \lim_{t\to\infty} \int_{k0}^{k(t)} u'(\chi y)\mathrm{d}y \exp[-\rho t] \\ &= \lim_{t\to\infty} \frac{\chi^{-\theta}}{1-\theta} \{k^{1-\theta} - k_0^{1-\theta}\} \exp[-\rho t] \\ &= \lim_{t\to\infty} \frac{\chi^{-\theta}}{1-\theta} k_0^{1-\theta} \{\exp[(1-\theta)\frac{(A-\delta-\rho n)t}{n\theta-(n-1)}] - 1\} \exp[-\rho t] \quad . \end{aligned}$$

This is fulfilled for

$$(1-\theta)\frac{(A-\delta-\rho n)}{n\theta - (n-1)} - \rho < 0 \quad ,$$

and using (43) it requires that

$$\frac{(1-\theta)}{\theta}(A-\delta) - \frac{\rho}{\theta} < 0,$$

which is fulfilled because of (38).

Insertion of (44) and (37) in (2) provides (46) and condition (45) has to hold for positive growth. □

THEOREM 4.3. *There exists a set of feasible parameter specifications for* $A, \delta, \rho, \theta, n$ *which enables an economy with secure property rights to grow forever but not an otherwise identical economy without property rights. The possibility for this scenario increases with the number of groups.*

Proof. Conditions (39) and (45) have to hold for positive growth with secure property rights and without property rights, respectively. Condition (45) can always be violated by a sufficiently large n, whereas (39) is independent from n. □

THEOREM 4.4. *If an economy is capable of long-run growth without property rights then growth in an otherwise identical economy with secure property rights is higher. The difference in growth rates is given by*

$$\Delta\gamma = \varphi \frac{n-1}{n\theta - (n-1)} \ , \tag{47}$$

which is increasing in the number of competing groups.

Proof. Substraction of (46) from (42) provides (47). The difference increases in n since $\partial(\Delta\gamma)/\partial n = \theta \left[1 - n/1 - \theta)\right]^{-2} > 0$. □

In [13] it has been discussed under which conditions the model may display a voracity effect.

DEFINITION 4.1. *Voracity-Effect[13]: Optimal adjustment of consumption behavior after a positive productivity shock leads to a lower growth rate of the economy.*

The following theorem may shed a new light on the finding in [13].

THEOREM 4.5. *If an economy with secure property rights is capable of long-run growth and a feedback Nash-equilibrium for an otherwise identical economy without property rights exists, then there exists no voracity effect.*

Proof. From (46) we obtain $\partial(\dot{k}/k)/\partial A = (n\theta - (n-1))^{-1}$, which is positive because of (43). □

In conclusion, a voracity effect may only exist if there exists no balanced growth path with secure property rights (economic conditions are too favorable, $\varphi < 0$, and utility grows without bound), and the intertemporal elasticity of substitution is very high, $\theta < (n-1)/n$, so that utility in an economy without property rights is still bounded. In other words, the existence of a balanced growth path in an economy with secure property rights excludes the existence of a voracity effect without property rights.

TABLE 2

Growth Rates Without Property Rights

Relative to Growth Rates With Property Rights

	$n=2$	$n=3$	$n=5$	$n=20$
Basic[a]	51	31	14	$--$
$\rho=0.04$	43	20	$--$	$--$
$A=0.5$	54	36	20	0.8
$\delta=0.1$	48	28	9	$--$
$\theta=8$	47	28	12	$--$

[a] $A=0.25$, $\rho=0.02$, $\theta=4$, $\delta=0.05$. Numbers in percent and rounded.

The question remains whether the absence of property rights lowers economic growth *significantly.* For that purpose we calculate growth rates for parameterized economies with and without property rights. The results are presented in Table 2 as the growth rate without property rights relative to the growth rate with property rights (in percent). The basic scenario assumes a capital output ratio of $1/A=4$. The results are especially interesting with respect to the corresponding findings from Table 1. If the possibility of long-run growth exists, the relative performance of an economy without property rights is much worse than in a neoclassical economy. If only two competing groups exist, the growth rate is about half that of an economy with secure property rights. If n rises up to three this ratio reduces to about one third. A slightly further increase of n may already produce disaster. The $--$ Symbol in Table 2 reflects that condition (45) is not fulfilled: Although a feedback equilibrium exists it implies a negative capital growth rate. Hence the economy converges towards the origin.

5 Development Dynamics

In this section we combine both models of the proceeding sections by introducing the convex growth technology

$$f(k)=Ak+Bk^{\alpha}, \quad A,B>0 \;, 0<\alpha<\theta. \tag{48}$$

The model for a competitive economy with secure property rights has first been presented in [9]. Our discussion is related to the textbook presentation in [3]. The advantage of the new technology is that the growth model displays transitional dynamics and we can show the existence of conditional convergence of growth rates, where the condition is the existence of secure property rights.

Since the growing economy has no steady-state in c and k, we first introduce the consumption capital ratio $\chi = c/k$ as control-like variable and the output capital ratio as $z = f(k)/k$ as state-like variable. Note that $z \geq A$ for $k \geq 0$.

Again, we first summarize the behavior of an economy with secure property populated by a continuum $[0, n]$ of price taking consumers.

THEOREM 5.1. *If*

$$\varphi > 0 \ , \tag{49}$$

$$A - \delta > \varphi \ , \tag{50}$$

then an economy with secure property rights develops along a unique adjustment path towards the saddlepoint equilibrium at

$$\chi = \frac{\varphi}{n} \ , \tag{51}$$

$$z = A \ , \tag{52}$$

with constant positive growth at the equilibrium.

The proof is in [3], Ch. 4.5.1.

THEOREM 5.2. *If an economy with secure property rights has a unique path of positive growth, and if*

$$\theta > \frac{n-1}{n} \ , \tag{53}$$

$$A - \delta > \varphi \frac{\theta n}{n\theta - (n-1)} \Leftrightarrow \frac{A - \delta}{n} > \rho \ , \tag{54}$$

then an otherwise identical developing economy without property rights has a unique path of positive growth towards the equilibrium

$$\chi^{\star} = \varphi \frac{\theta}{n\theta - (n-1)} \ , \tag{55}$$

$$z^\star = A \ . \tag{56}$$

Proof. Step 1: Using (48) system (20) and (21) can be rewritten in state- like-control-like notation as

$$\dot{z} = (\alpha-1)(z-A)\left[z-\delta-\frac{n\theta-(n-1)}{\theta}\chi\right] \ , \tag{57}$$

$$\dot{\chi} = \chi\left[-\frac{\theta-\alpha}{\theta}(z-A)-\varphi+\frac{n\theta-(n-1)}{\theta}\chi\right] \ . \tag{58}$$

Since we have assumed that $\theta > \alpha$, conditions(53) and (54) ensure that a unique positive equilibrium for $z \geq A$ exists. It is located at $\chi^\star, z^\star$. The Jacobian determinant of (57) and (58) evaluated at the equilibrium is $\varphi(\alpha-1)(A-\delta-\varphi)$, and hence negative from (53) and (54). The equilibrium is a saddlepoint. Figure 3 shows the (relevant part of the) phase diagram.

We next show that the transversality condition (18) is fulfilled. Using $\chi = c/k$, it follows that

$$\int_{k0}^{k} u'(c(y))\mathrm{d}y = \int_{k0}^{k} u'(y\chi(y))\mathrm{d}y.$$

Since χ converges, it is bounded and with $\dot{\chi} < 0$ the minimum can be written as

$$\chi^* = \min\{\chi(t), t \in [0,\infty)\} > 0.$$

With $u'' < 0$ it follows that

$$\int_{k0}^{k} u'(y\chi(y))\mathrm{d}y \ \ \leq \ \ \int_{k0}^{k} u'(y\chi^*)\mathrm{d}y = (\chi^*)^{-\theta}\int_{k0}^{k} y^{-\theta}\mathrm{d}y.$$

Integration by substitution provides

$$\int_{k(0)}^{k(t)} y^{-\theta}\mathrm{d}y \ \ = \ \ \int_0^t k(s)^{-\theta}\dot{k}(s)ds \leq \int_0^t k(s)^{-\theta-1}(z_0-\delta-n\chi^*)ds,$$

since $\dot{z} < 0$.

From $\gamma_k := \dot{k}/k = A + Bk^{\alpha-1} - \delta - n\chi$ it follows that $\dot{\gamma_k} = B(\alpha-1)k^{\alpha-2} - n\dot{\chi} < 0$ and therefore $k(t) \geq k_0 \exp(\gamma^* t)$ with $\gamma^* = A - \delta - n\chi^*$.

Equation (18) can now be rewritten as

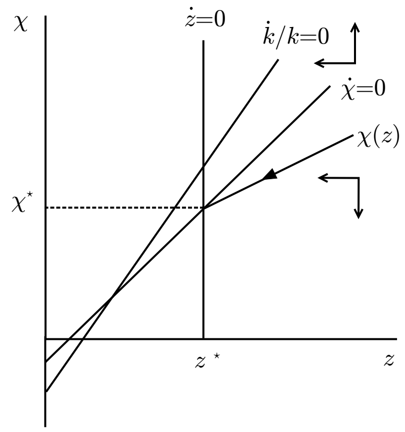
$$\begin{aligned}
&\lim_{t\to\infty}\int_{k0}^{k} u'(c(y))\mathrm{d}y\exp[-\rho t] \le const * \lim_{t\to\infty}\int_0^t k(s)^{-\theta-1}\mathrm{d}s\exp[-\rho t].\\
\le\ & const * \lim_{t\to\infty}\int_0^t \{k_0\exp[(A-\delta-\varphi\frac{n\theta}{n\theta-(n-1)})s]\}^{-(\theta+1)}\mathrm{d}s\exp[-\rho t]\\
=\ & const * \lim_{t\to\infty}\exp[-((A-\delta-\varphi\frac{n\theta}{n\theta-(n-1)})(\theta+1)-\rho)t],
\end{aligned}$$

which equals zero because (54) guarantees that

$$-(A-\delta-\varphi\frac{n\theta}{n\theta-(n-1)})(\theta+1)-\rho<0.$$

Hence, $\lim_{t\to\infty}\int_{k0}^{k} u'(c(y))\mathrm{d}y\exp[-\rho t]=0$ because with increasing $k(t)$, the integral cannot become negative.

Figure 3: Phase Diagram: Endogenous Growth Without Property Rights



Step 2: It remains to prove that the consumption strategy $\chi(z)$ is compatible with positive long-run growth. After insertion of (48) into (2) the *real* growth rate of the economy is obtained in state-like control-like notation as $\gamma_k = z-\delta-n\chi$. For positive growth the $\chi(z)$ curve must be situated below the *real* $\dot{k}/k=0$-locus

$$\tilde{\chi}(z)=(z-\delta)/n\ . \tag{59}$$

Since $\tilde{\chi}(z^\star)=(A-\delta)/n$, condition (54) ensures that χ lies below the $\dot{k}/k=0$-locus at the equilibrium. The $\dot{k}/k=0$-curve is linear with slope $1/n$.

Assume $\chi(z)$ has intersections with the $\dot{k}/k = 0$-locus for $z > A$. With (57), (58) and (59), the slope $\chi'(z) = \dot{\chi}/\dot{z}$ at the intersection points is given by

$$\chi'(z) = \frac{-\frac{\theta-\alpha}{\theta}(z-A) - \varphi + \frac{n\theta-(n-1)}{\theta n}(z-\delta)}{(\alpha-1)(z-A)\frac{n-1}{\theta}} \ . \tag{60}$$

Let the intersection point closest to $z^\star$ for $z > A$ be denoted by z_1. Then the slope of $\chi(z)$ in z_1 has to be larger than $1/n$: $\chi'(z_1) > 1/n$. This leads to the inequality

$$z_1 < \frac{A\left[(1-\alpha)\frac{n-1}{n\theta} - \frac{\theta-\alpha}{\theta}\right] + \varphi + \left[\frac{n\theta-(n-1)}{n\theta}\right]\delta}{\alpha/(n\theta)} \ . \tag{61}$$

With insertion of φ from (40) this simplifies to

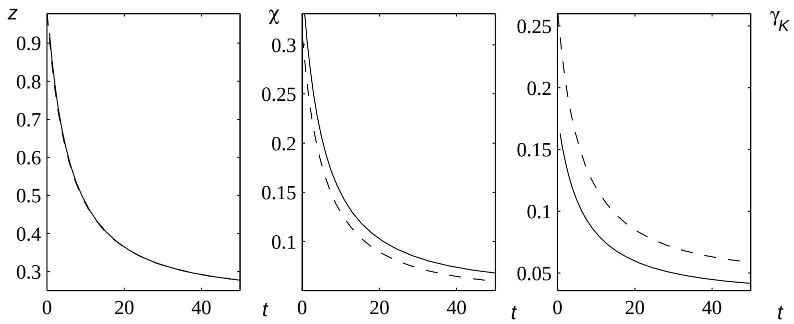
$$z_1 < \frac{A(\alpha-1) + \delta + n\rho}{\alpha} < A \ , \tag{62}$$

where the last inequality follows from (54) . This is a contradiction to the assumption $z_1 > A$. □

Note that the conditions for long-run growth without property rights are just the same as in the linear model. The pessimistic findings of the previous section are not caused by the linearity assumption but an inherent feature of feedback consumption strategies in an economy with growth potential but without property rights. Since both models display the same long-run solution, all results from Theorem 4.3 – Theorem 4.5 are also results of a comparative steady-state analysis for the non-linear model.

Now compare development dynamics with and without property rights. We use the basic parameterization from Table 2 and $B = 1$, $\alpha = 0.3$. The implied steady- state capital output ratio is $1/A = 4$ and may represent a fully developed country. Again we employ the method of backward integration and terminate at $z = 1$, i.e. at a capital output ratio of one, which may represent a less developed country. After having obtained the stable manifold and reverting the solution sequence to forward looking we calculate the *real* adjustment path by employing the trapezoidal rule and the *real* growth rate $\gamma_k = z - \delta - 2\chi$ (See Section 3 for details). We compare the development path to the solution for an otherwise identical economy with property rights.

Figure 4: Development Dynamics: Endogenous Growth



Solid Lines: Without Property Rights, Dashed Lines: With Property Rights

$n = 2$, $\alpha = 0.3$, $A = 0.25$, $B = 1$, $\theta = 4$, $\rho = 0.02$.

In Figure 4 solid lines show adjustment dynamics without property rights and dashed lines the corresponding development path with property rights. Without property rights people select a higher consumption capital ratio during the adjustment process and in the long–run. During the adjustment process the capital output ratio, $1/z$, is lower without property rights. The economy with property rights shows a *temporarily* higher capital output ratio and arrives at a steady-state of *permanently* higher growth in capital and consumption. Although both economies arrive at the same (gross) capital productivity, $A - \delta$, the investment rate, $I/K = z - n\chi$, is higher in the economy with property rights. In our parameterized example the investment rate is 30 percent higher if property rights are secure.

6 Conclusion

We have argued that established and enforceable property rights are an important prerequisite for successful development. The neoclassical growth model as well as the convex model of growth can explain conditional convergence. The condition is the existence of secure property rights. If the technology allows for long-run growth, insecure property rights lead to a lower rate of investment and adjustment dynamics towards a steady-state of lower growth compared to an otherwise identical economy with secure property rights.

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